Economic Growth, Capital Accumulation and Output

GRADUATE MACRO – LAB SESSION 12

ETTORE GALLO

Class Outline

1. **The Long Run**
2. **Ch. 10: The Facts of Growth**

   10-1 Measuring the Standard of Living

   10-2 Growth in Rich Countries since 1950

   10-4 Thinking about Growth: A Primer
3. **Ch. 11: Saving, Capital Accumulation, and Output (Solow Model)**

   11-1 Interactions between Output and Capital

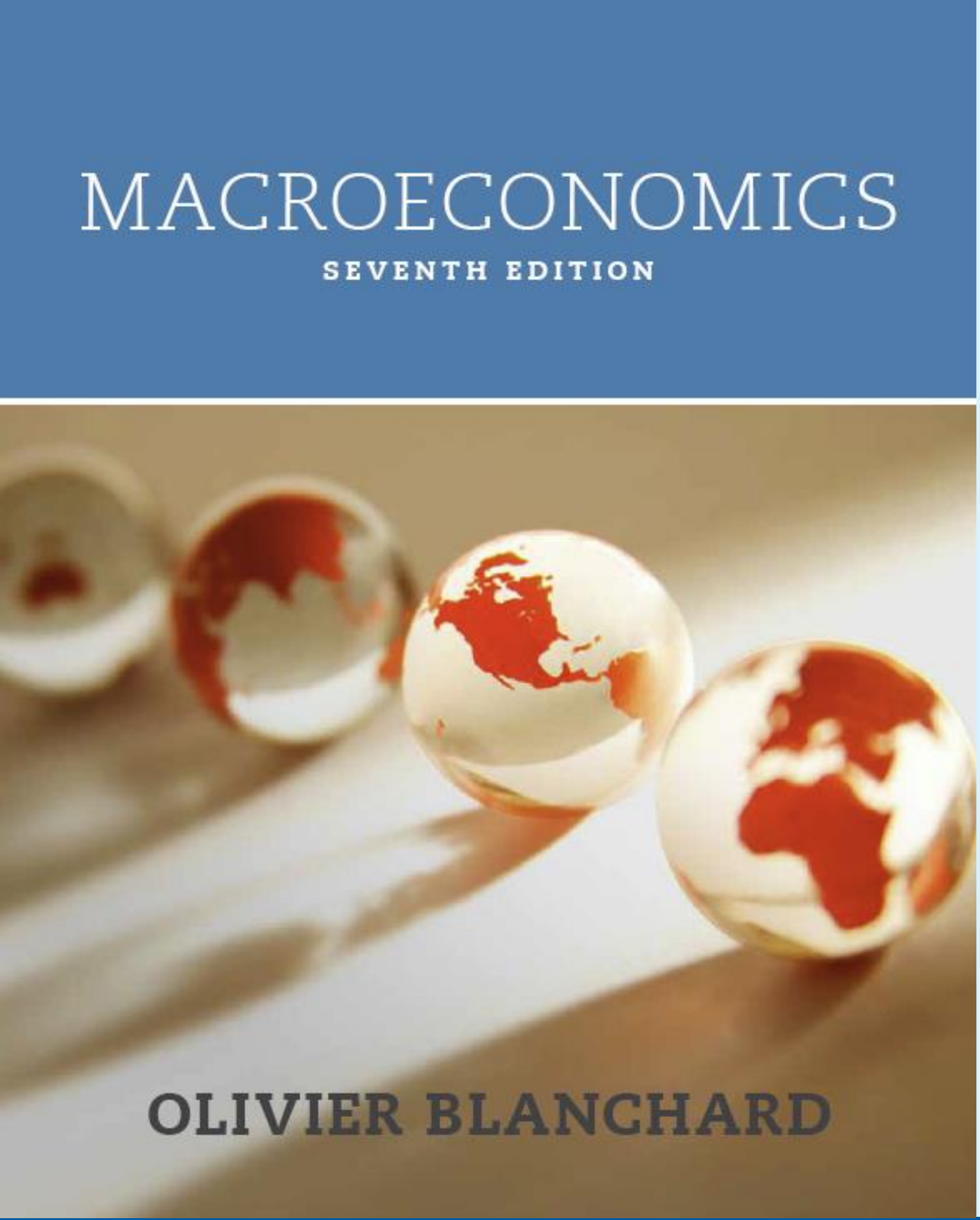
   11-2 The Implications of Alternative Saving Rates

   11-3 Getting a Sense of Magnitudes

   11-4 Physical versus Human Capital

Short vs. medium run vs. **long run**

- In the **short run**, the factors that determine movements in output are the factors we focused on in the preceding chapters: monetary policy, fiscal policy, and so on.
- In the **medium run**, output tends to return to the natural level. The factors that determine unemployment and, by implication, output, are of key interest in the medium run
- Over the **long run,** all markets tend to remain in equilibrium (both prices and quantities adjust). Therefore, the analysis focuses on economic growth, labor productivity and technology.
  - In the long run, saving does not affect growth, but does affect the level of output per worker.



The Facts of Growth

Chapter 10

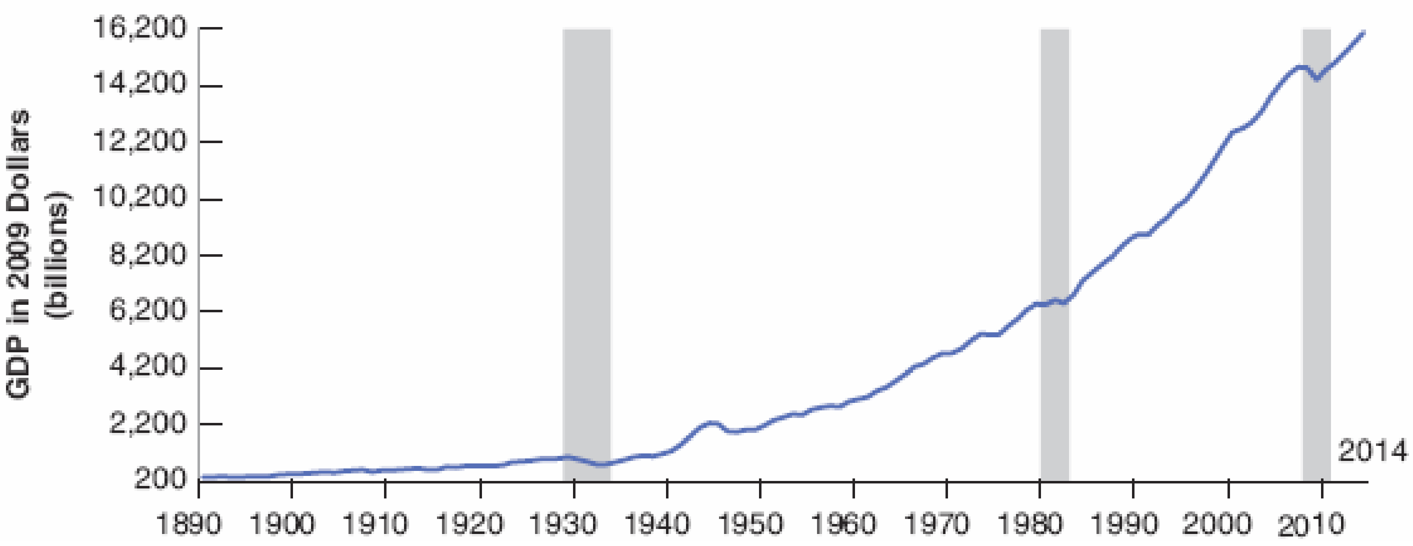
The Facts of Growth

- **Growth** is the steady increase in aggregate output over time.
- We now shift our focus from economic fluctuations and the determination of output in the *short and medium run* to growth and the determination of output in the *long run*.

Figure 10-1 U.S. GDP since 1890. U.S. GDP per person since 1890

Panel *A*

Source: 1890–1947: *Historical Statistics of the United States.* http://hsus.cambridge.org/HSUSWeb/toc/hsusHome.do. 1948 to 2014: *National Income and Product Accounts*. Population estimates 1890 to 2014, from Louis Johnston and Samuel H. Williamson, "What Was the U.S. GDP Then?" Measuring Worth, 2015, https://www.measuringworth.com/datasets/usgdp/

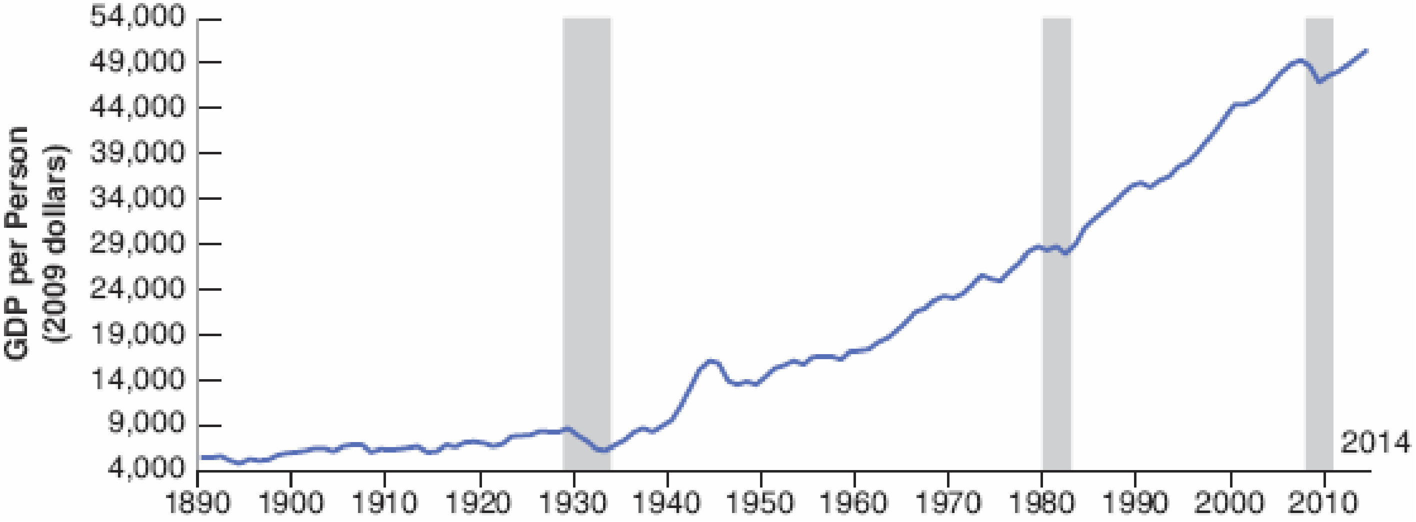


Panel *A* shows the enormous increase in U.S. output since 1890, by a factor of 46.

Figure 10-1 U.S. GDP since 1890. U.S. GDP per person since 1890 (cont'd)

Panel *B*

Source: 1890–1947: *Historical Statistics of the United States*. http://hsus.cambridge.org/HSUSWeb/toc/hsusHome.do. 1948 to 2014: *National Income and Product Accounts*. Population estimates 1890 to 2014, from Louis Johnston and Samuel H. Williamson, "What Was the U.S. GDP Then?" Measuring Worth, 2015, https://www.measuringworth.com/datasets/usgdp/



Panel *B* shows that the increase in output is not simply the result of the large increase in U.S. population from 63 million to more than 300 million over this period. Output per person has risen by a factor of 9.

10-1 Measuring the Standard of Living

- We care about growth because we care about the standard of living.
- Output per person, rather than output itself, is the variable we compare over time or across countries.
- We need to correct for variations in exchange rates and systematic differences in prices across countries.
- When comparing the standard of living across countries, we use **purchasing power parity (PPP)** numbers which adjust the numbers for the **purchasing power** of different countries.
- The right measure on the production side is *output per worker* or *output per hour worked.*

10-2 Growth in Rich Countries since 1950

Table 10-1 The Evolution of Output per Person in Four Rich Countries since 1950

	Annual Growth Rate Output per Person (%)	Real Output per Person (2005 dollars)		
	1950–2011	1950	2011	2011/1950
France	2.5	6,499	29,586	4.6
Japan	4.1	2,832	31,867	11.3
United Kingdom	2.0	9,673	32,093	3.3
United States	2.0	12,725	42,244	3.3
Average	2.4	7,933	33,947	4.3

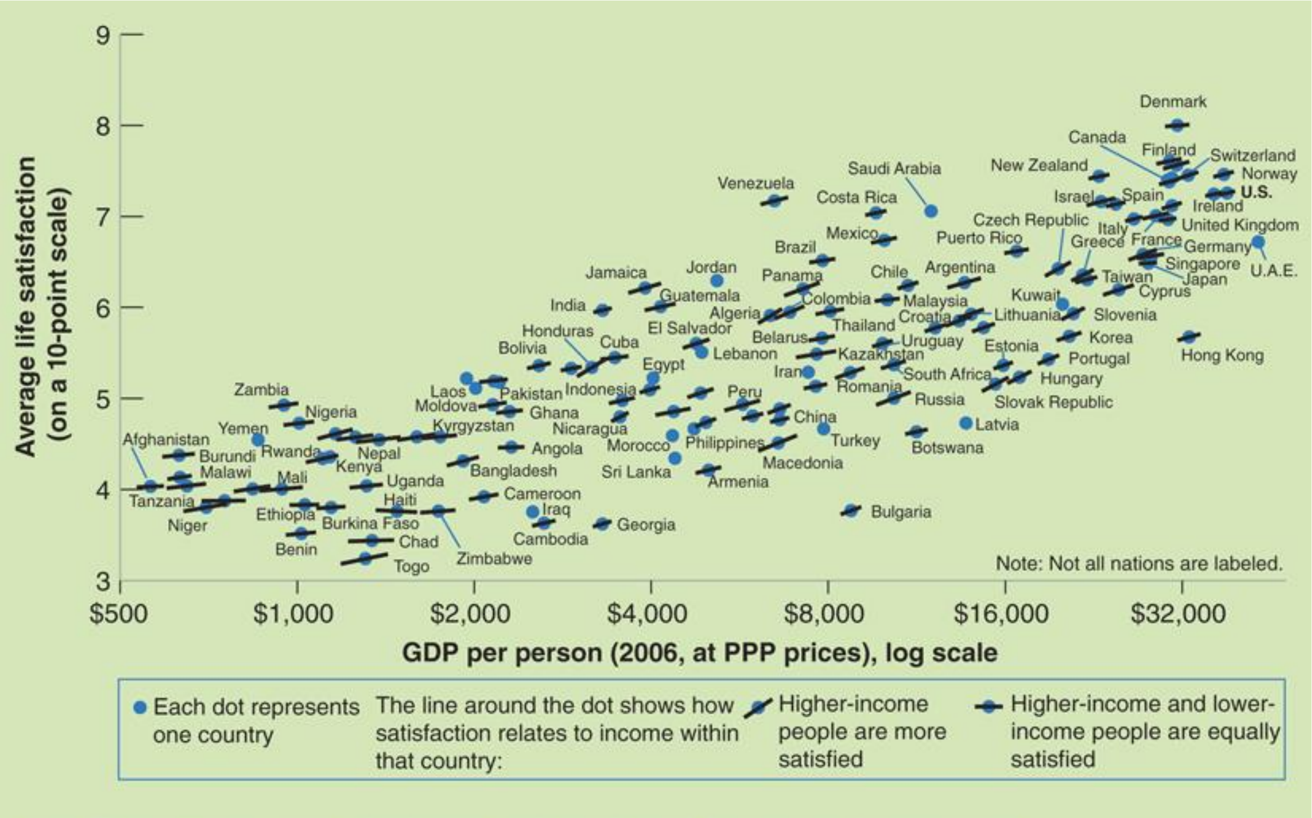
Notes: The data stop in 2011, the latest year (at this point) available in the Penn tables.
The average in the last line is a simple unweighted average.

Source: Penn Tables. http://cid.econ.ucdavis.edu/pwt.html

- There has been a large increase in output per person due in part to the **force of compounding**.
- There has been a **convergence** of output per person across countries.

FOCUS: Does Money Lead to Happiness?

Figure 1 Life Satisfaction and Income per Person



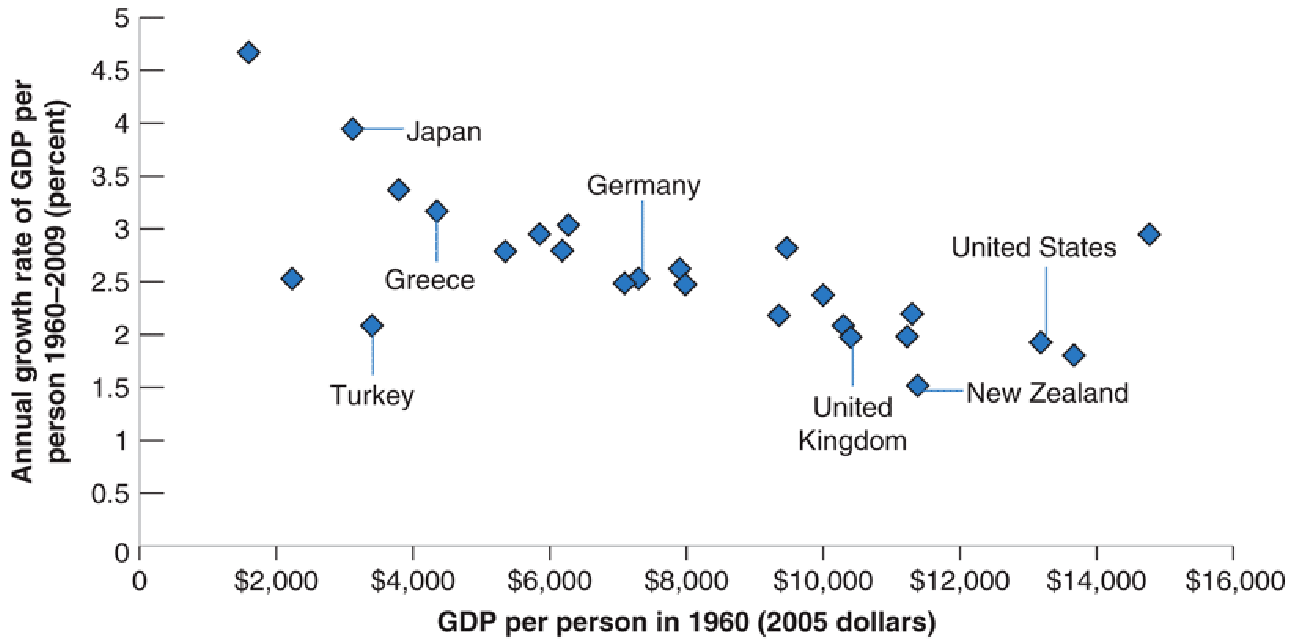
Source: Betsey Stevenson and Justin Wolfers, Wharton School at the University of Pennsylvania.

- **Easterlin paradox:** Once basic needs are satisfied, higher income per person does not increase happiness, and the level of income relative to others, rather than the absolute level of income, matters

10-2 Growth in Rich Countries since 1950

Figure 10-2 Growth Rate of GDP Per Person since 1950 versus GDP per Person in 1950; OECD Countries

Countries with lower levels of output per person in 1950 have typically grown faster.



Source: See Table 10-1.

10-4 Thinking About Growth: A Primer

- Aggregate production function *F*:

$$Y = F(K, N) \tag{10.1}$$

  where *Y* is output, *K* is capital, and *N* is labor.
- The function *F* depends on the **state of technology**.
- **Constant returns to scale**:

$$xY = F(xK, xN) \tag{10.2}$$

- **Decreasing returns to capital:** Increases in capital lead to smaller and smaller increases in output.
- **Decreasing returns to labor:** Increases in labor lead to smaller and smaller increases in output.

10-4 Thinking About Growth: A Primer

- The production function and constant returns to scale imply a simple relation between output per worker (*Y/N*) and capital per worker (*K/N*):

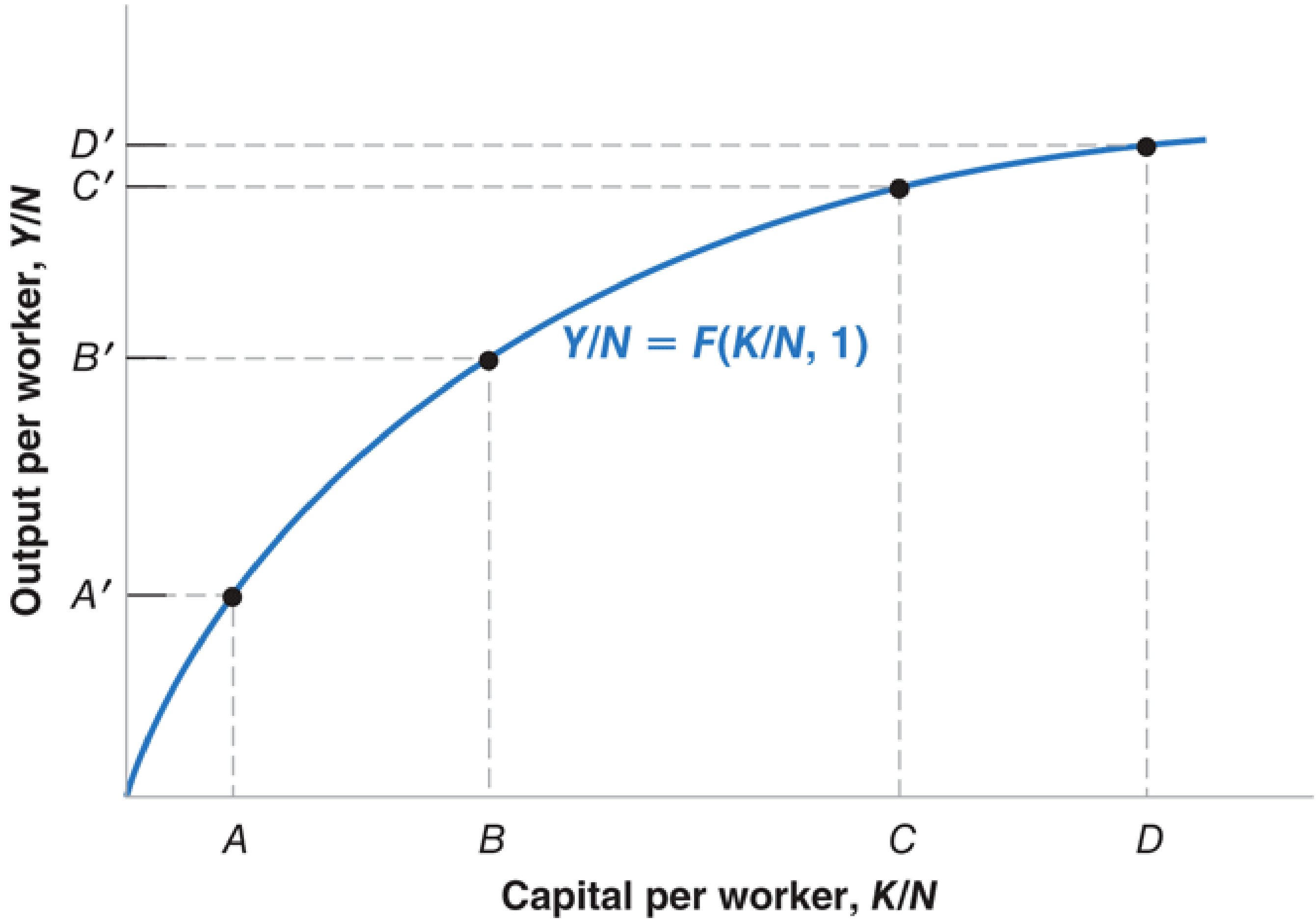
$$\frac{Y}{N} = F\left(\frac{K}{N}, \frac{N}{N}\right) = F\left(\frac{K}{N}, 1\right) \tag{10.3}$$

- Increases in capital per worker: Movements along the production function.
- Improvements in the state of technology: Shifts (up) of the production function.
- Growth comes from **capital accumulation** (a higher **saving rate**) and **technological progress** (the improvement in the state of technology).

10-4 Thinking About Growth: A Primer

Figure 10-4 Output and Capital per Worker

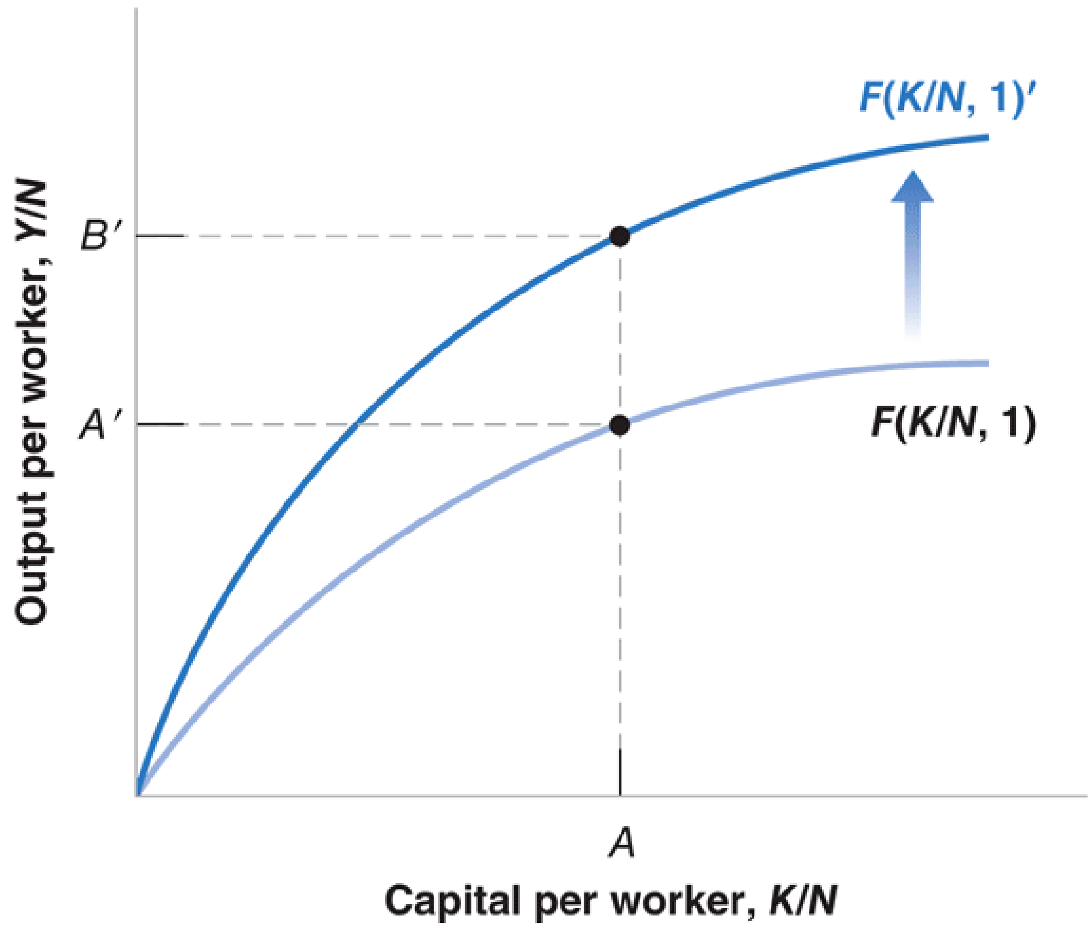
Decreasing returns to capital: Increases in capital per worker lead to smaller and smaller increases in output per worker.

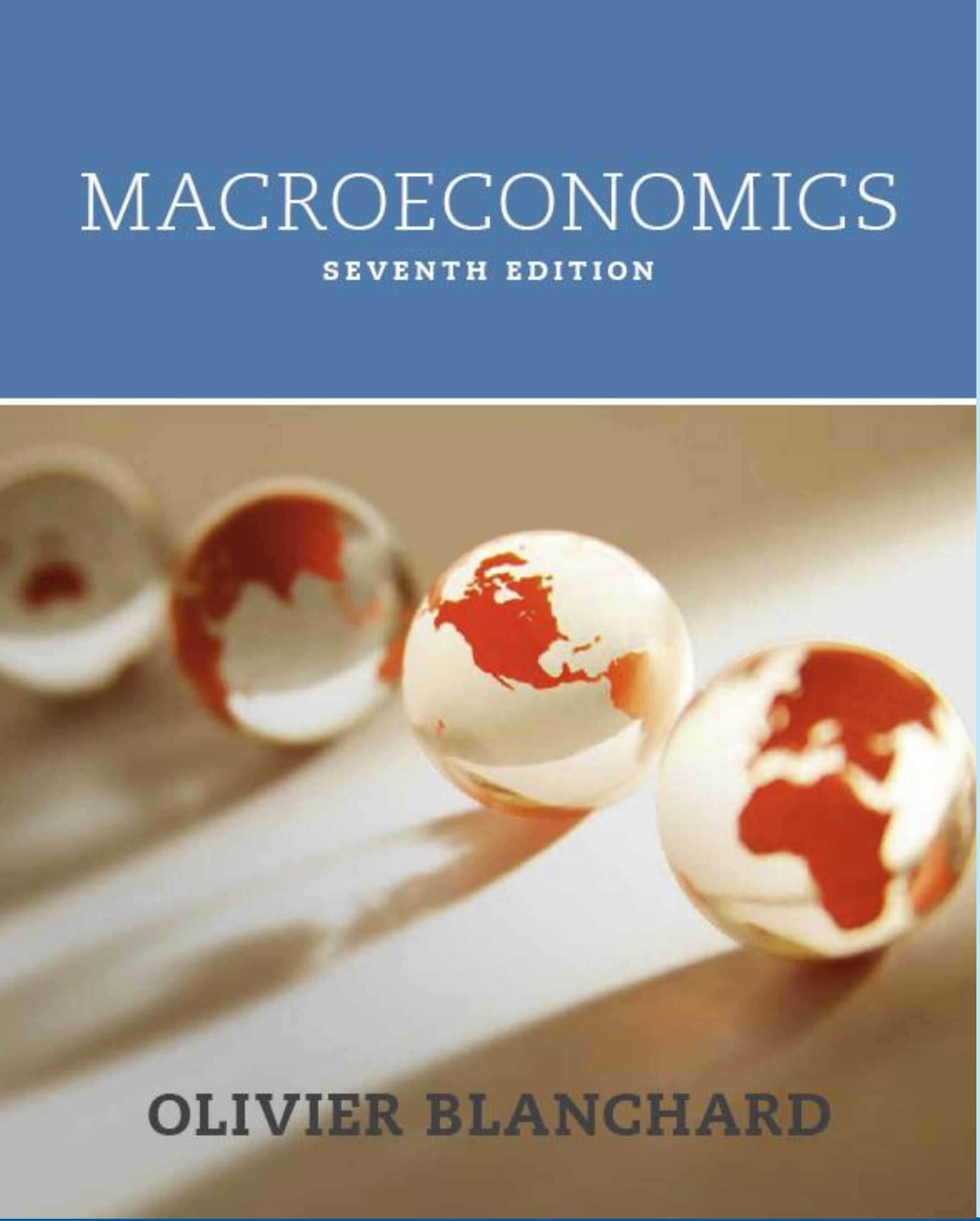


10-4 Thinking About Growth: A Primer

Figure 10-5 The Effects of an Improvement in the State of Technology

An improvement in technology shifts the production function up, leading to an increase in output per worker for a given level of capital per worker.





Saving, Capital Accumulation, and Output

Chapter 11

Saving, Capital Accumulation, and Output

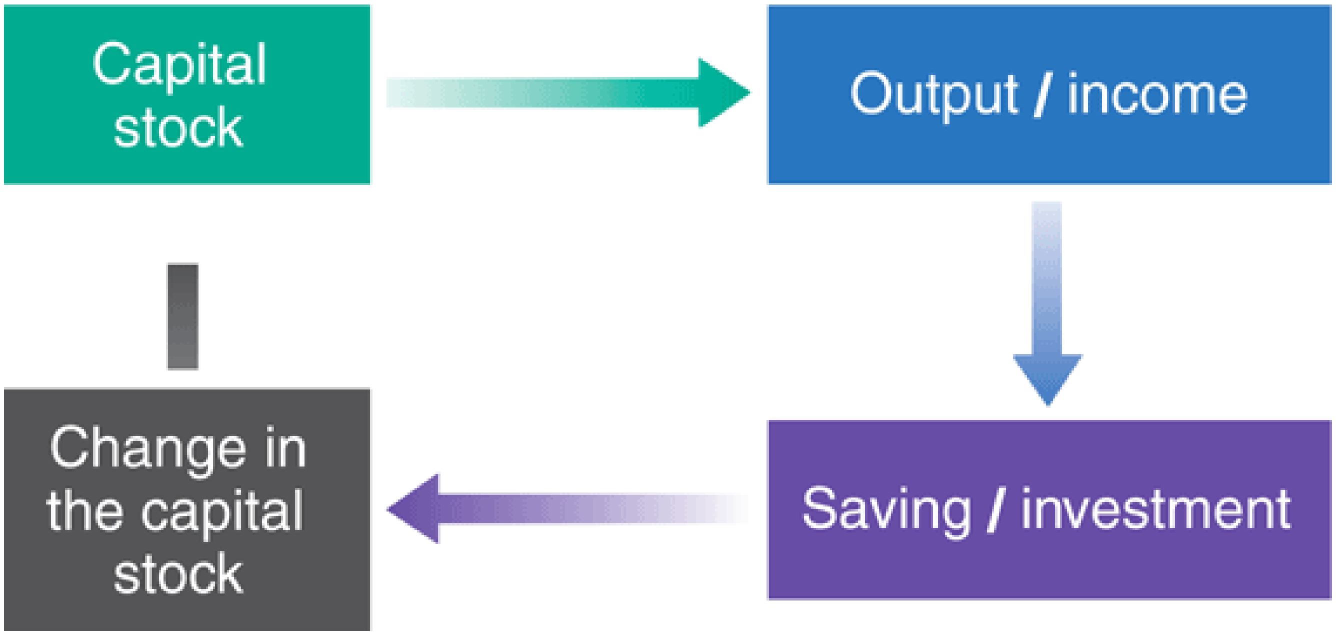
- Since 1970, the U.S. saving ratio—the ratio of saving to gross domestic product—has averaged only 17%, compared to 22% in Germany and 30% in Japan.
- Even if a lower saving rate does not permanently affect the growth rate, it does affect the level of output and the standard of living.

11-1 Interactions between Output and Capital

- Output in the long run depends on two relations:
  - The amount of capital determines the amount of output
  - The amount of output being produced determines the amount of saving, which in turn determines the amount of capital being accumulated over time

11-1 Interactions between Output and Capital

Figure 11-1 Capital, Output, and Saving/Investment



11-1 Interactions between Output and Capital

- Recall Chapter 10:

$$\frac{Y}{N} = F\left(\frac{K}{N}, 1\right) \quad \text{or} \quad f\left(\frac{K}{N}\right) \equiv F\left(\frac{K}{N}, 1\right)$$

- Assume that *N* is constant, and there is no technological progress, so *f* does not change over time:

$$\frac{Y_t}{N} = f\left(\frac{K_t}{N}\right) \tag{11.1}$$

- Higher capital per worker leads to higher output per worker.

11-1 Interactions between Output and Capital

- Assume:
  - The economy is closed: $I = S + (T - G)$
  - Public saving $(T - G)$ is 0: $I = S$
  - Private saving is proportional to income: $S = sY$
- So the relation between output and investment:

$$I_t = sY_t$$

- Investment is proportional to output.
- The higher (lower) output is, the higher (lower) is saving and so the higher (lower) is investment.

11-1 Interactions between Output and Capital

- The evolution of the capital stock is:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

- Replace investment by the above expression and divide both sides by *N*:

$$\frac{K_{t+1}}{N} = (1 - \delta)\frac{K_t}{N} + s\frac{Y_t}{N}$$

or

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N} \qquad (11.2)$$

- The change in the capital stock per worker is equal to saving per worker minus depreciation.

11-2 The Implications of Alternative Saving Rates

- Combining equations (11.1) and (11.2):

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s f\left(\frac{K_t}{N}\right) - \delta \frac{K_t}{N} \qquad (11.3)$$

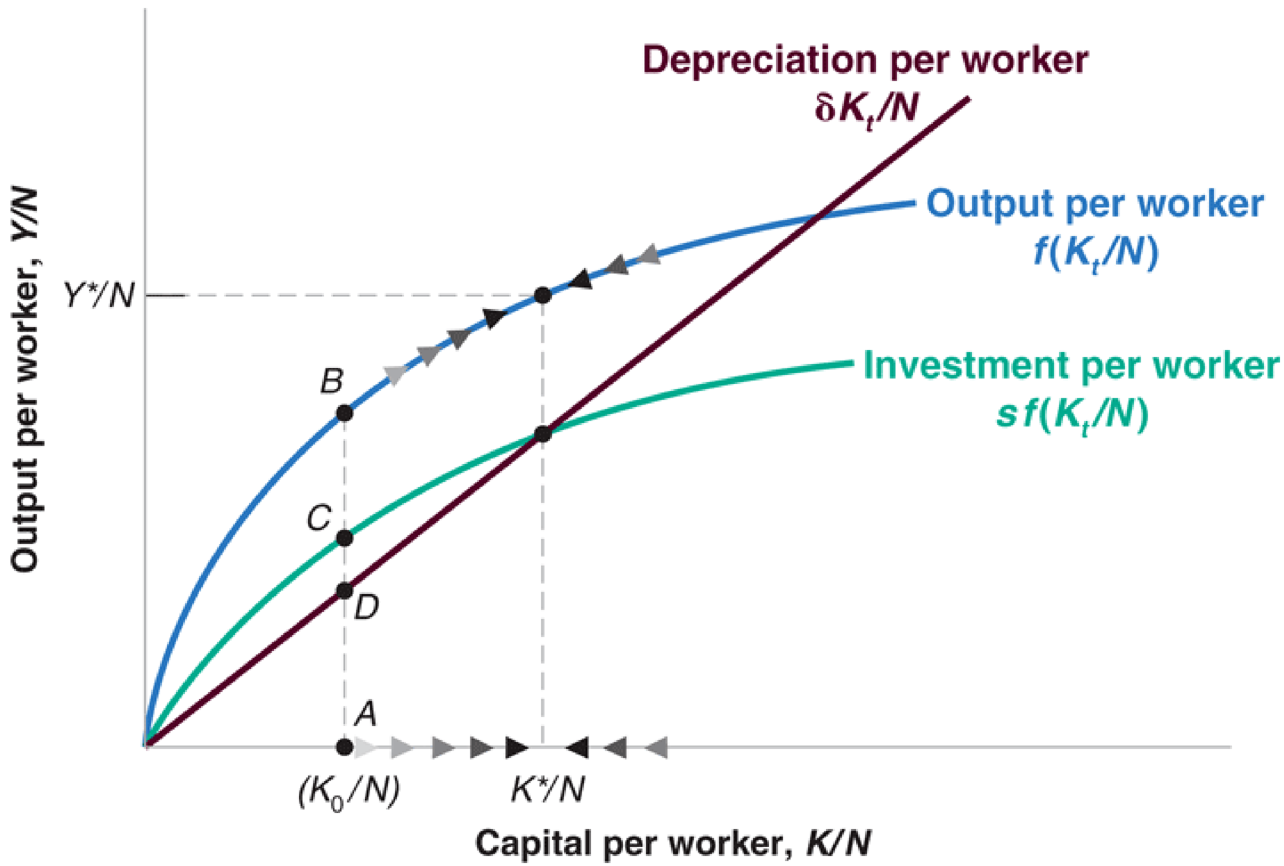
change in capital from year t to year $t+1$ = Invesment during year t − depreciation during year t

- If investment per worker exceeds (is less than) depreciation per worker, the change in capital per worker is positive (negative).

11-2 The Implications of Alternative Saving Rates

Figure 11-2 Capital and Output Dynamics

When capital and output are low, investment exceeds depreciation and capital increases. When capital and output are high, investment is less than depreciation and capital decreases.



11-2 The Implications of Alternative Saving Rates

- The state in which output per worker and capital per worker are no longer changing is called the **steady state** of the economy.

$$s f\left(\frac{K^*}{N}\right) = \delta \frac{K^*}{N} \qquad (11.4)$$

- The steady-state value of capital per worker is such that the amount of saving per worker is sufficient to cover depreciation of the capital stock per worker.

$$\frac{Y^*}{N} = f\left(\frac{K^*}{N}\right) \qquad (11.5)$$

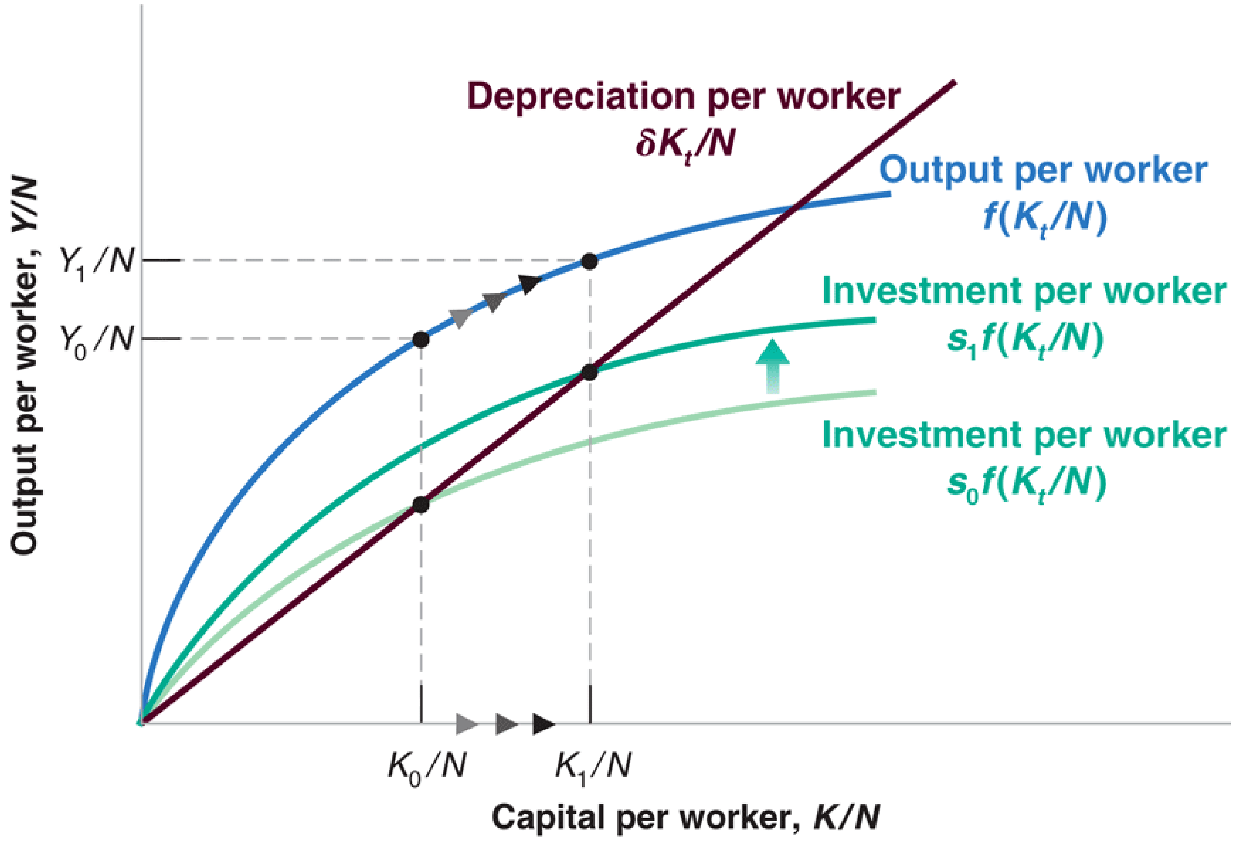
11-2 The Implications of Alternative Saving Rates

- *The saving rate has no effect on the long-run growth rate of output per worker, which is equal to zero.*
- *The saving rate determines the level of output per worker in the long run.*
- *An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever.*

11-2 The Implications of Alternative Saving Rates

Figure 11-3 The Effects of Different Saving Rates

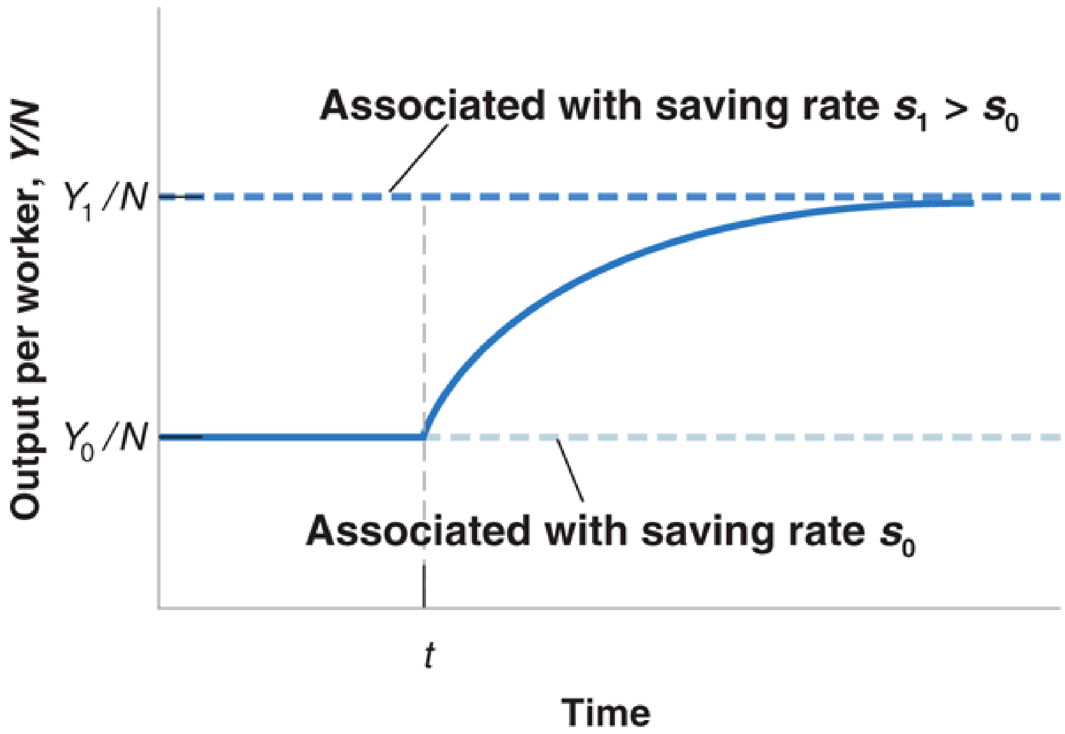
A country with a higher saving rate achieves a higher steady-state level of output per worker.



11-2 The Implications of Alternative Saving Rates

Figure 11-4 The Effects of an Increase in the Saving Rate on Output per Worker in an Economy Without Technological Progress

An increase in the saving rate leads to a period of higher growth until output reaches its new higher steady-state level.



11-2 The Implications of Alternative Saving Rates

Figure 11-5 The Effects of an Increase in the Saving Rate on Output per Worker in an Economy with Technological Progress

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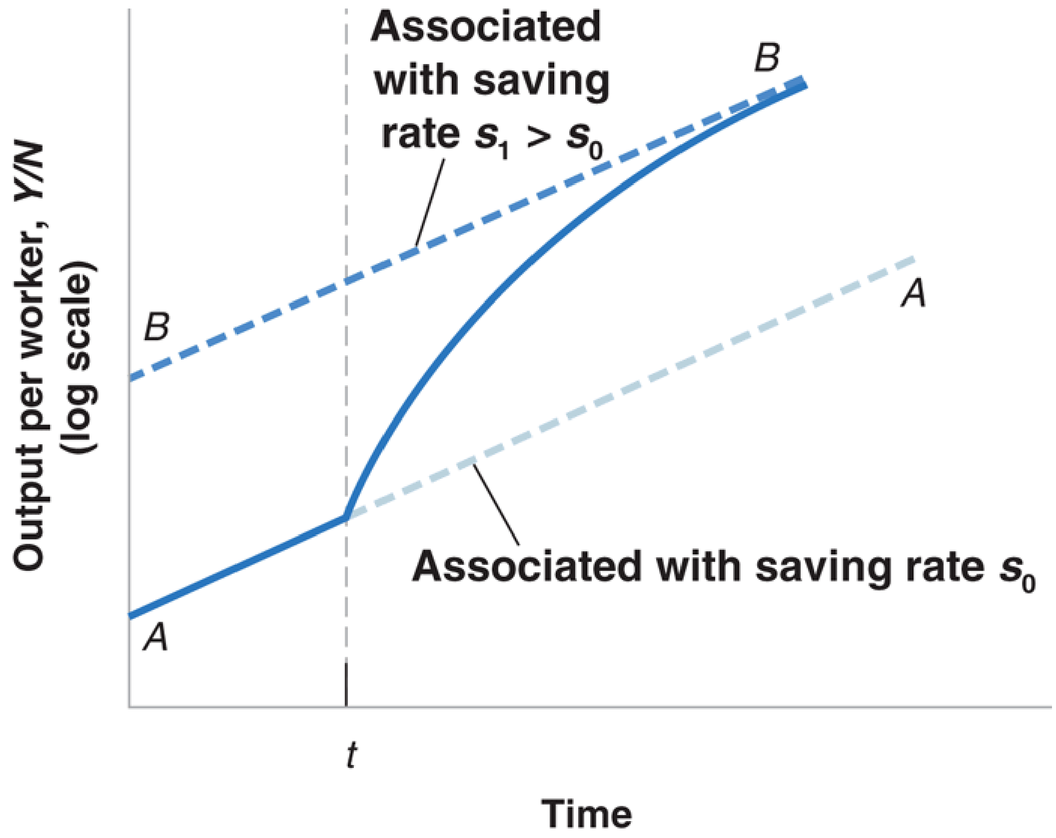


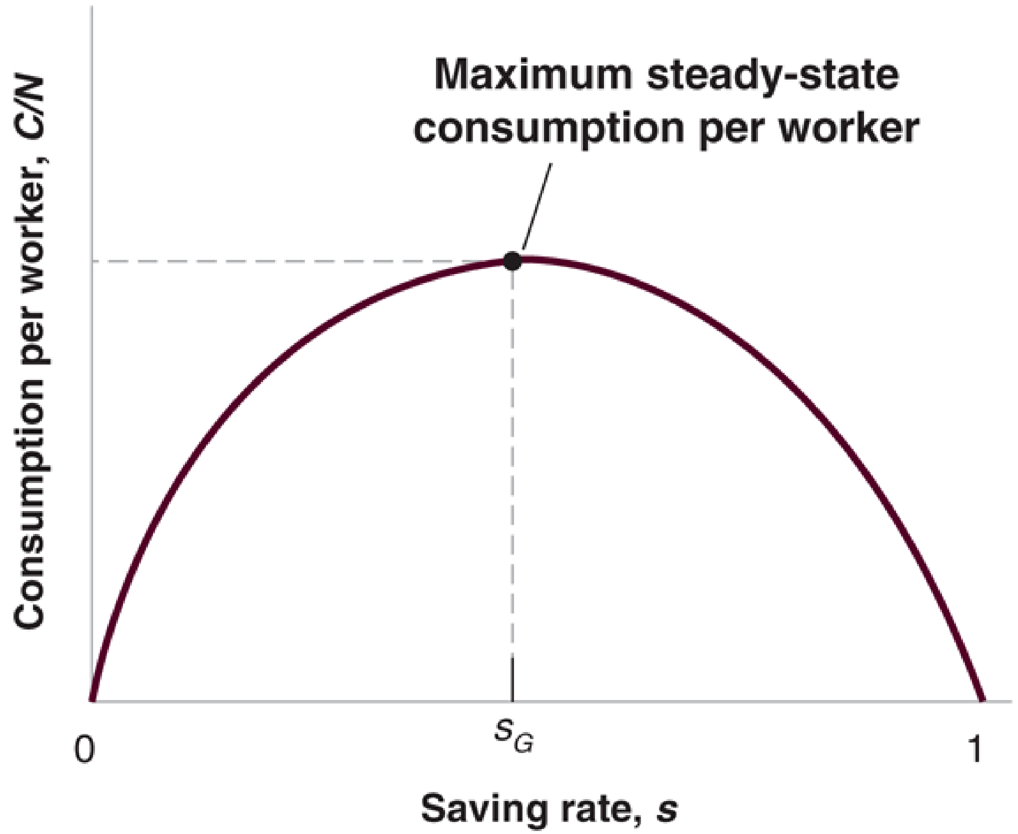
11-2 The Implications of Alternative Saving Rates

- What matters to people is not how many is produced, but how much they consume.
- Governments can affect the saving rate by:
  - changing public saving (budget surplus)
  - using taxes to affect private saving
- **Golden-rule level of capital:** The level of capital associated with the saving rate that yields the highest level of consumption in steady state.

11-2 The Implications of Alternative Saving Rates

Figure 11-6 The Effects of the Saving Rate on Steady-State Consumption per Worker

An increase in the saving rate leads to an increase, then to a decrease in steady-state consumption per worker.



11-2 The Implications of Alternative Saving Rates

- For a saving rate between zero and the golden-rule level, a higher saving rate leads to higher capital per worker, higher output per worker and higher consumption per worker.
- For a saving rate greater than the golden-rule level, a higher saving rate still leads to higher capital per worker and output per worker, but lower consumption per worker.
- An increase in the saving rate leads to lower consumption for some time but higher consumption later.

11-3 Getting a Sense of Magnitudes

- Assume the production function *f* (Cobb-Douglas):

$$Y = \sqrt{K}\sqrt{N} \tag{11.6}$$

- so that equation (11.3) becomes:

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\sqrt{\frac{K_t}{N}} - \delta\frac{K_t}{N} \tag{11.7}$$

- which describes the evolution of capital over time.

11-3 Getting a Sense of Magnitudes

- Equation (11.7) implies that capital per worker in the steady state (K^*/N) becomes:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2 \tag{11.8}$$

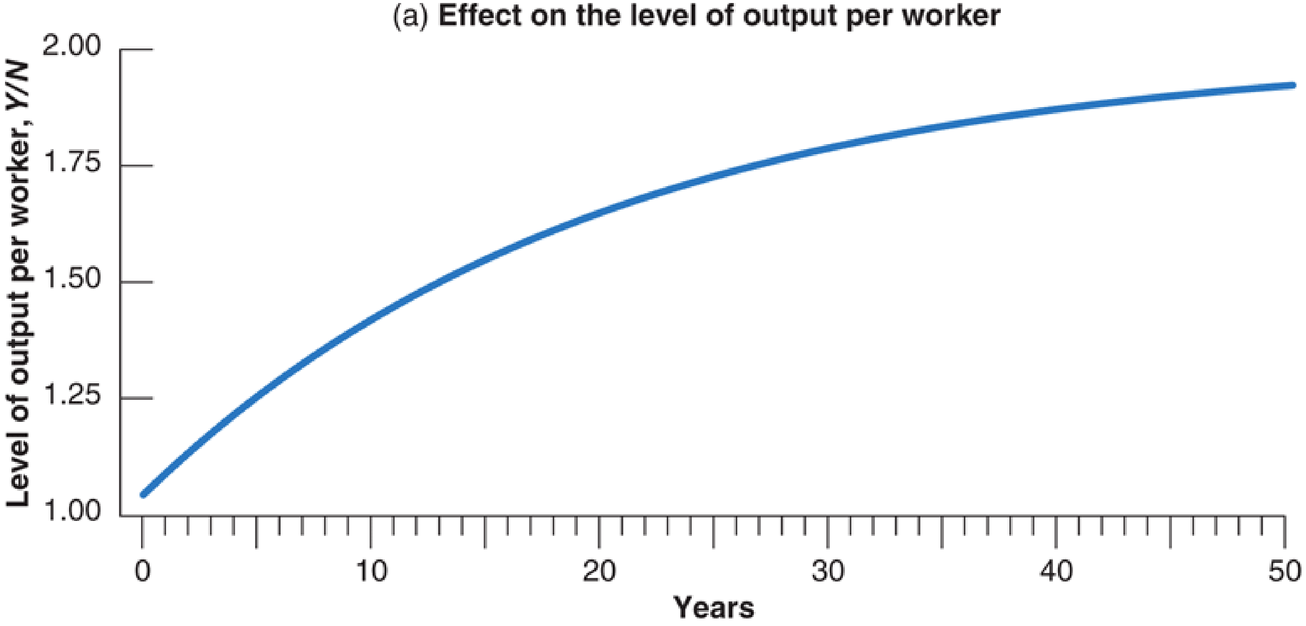
- Combining equations (11.6) and (11.8) gives the steady state output per worker:

$$\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}} = \sqrt{\left(\frac{s}{\delta}\right)^2} = \frac{s}{\delta} \tag{11.9}$$

- In the long run, output per worker doubles when the saving rate doubles.

11-3 Getting a Sense of Magnitudes

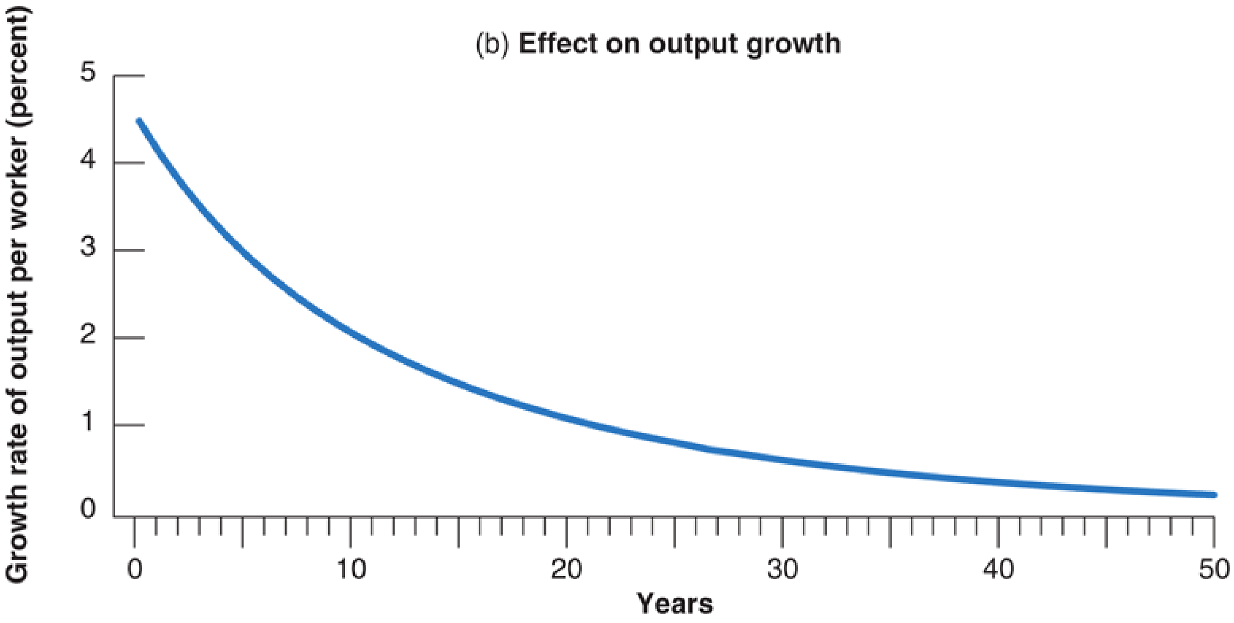
Figure 11-7(a) The Dynamic Effects of an Increase in the Saving Rate from 10% to 20% on the Level and the Growth Rate of Output per Worker



It takes a long time for output to adjust to its new higher level after an increase in the saving rate. Put another way, an increase in the saving rate leads to a long period of higher growth.

11-3 Getting a Sense of Magnitudes

Figure 11-7(b) The Dynamic Effects of an Increase in the Saving Rate from 10% to 20% on the Level and the Growth Rate of Output per Worker



It takes a long time for output to adjust to its new higher level after an increase in the saving rate. Put another way, an increase in the saving rate leads to a long period of higher growth.

11-3 Getting a Sense of Magnitudes

- In the steady state, consumption per worker is:

$$\frac{C}{N} = \frac{Y}{N} - \delta\frac{K}{N}$$

- Given equations (11.8) and (11.9), the steady-state consumption per worker is:

$$\frac{C}{N} = \frac{s}{\delta} - \delta\left(\frac{s}{\delta}\right)^2 = \frac{s(1 - s)}{\delta}$$

- Table 11-1 gives the steady-state values of capital per worker, output per worker and consumption per worker for different saving rates (given δ=10%)

11-3 Getting a Sense of Magnitudes

Table 11-1 The Saving Rate and the Steady-State Levels of Capital, Output, and Consumption per Worker

Saving Rate s	Capital per Worker K/N	Output per Worker Y/N	Consumption per Worker C/N
0.0	0.0	0.0	0.0
0.1	1.0	1.0	0.9
0.2	4.0	2.0	1.6
0.3	9.0	3.0	2.1
0.4	16.0	4.0	2.4
0.5	25.0	5.0	2.5
0.6	36.0	6.0	2.4
. . .	. . .	. . .	. . .
1.0	100.0	10.0	0.0

11-4 Physical versus Human Capital

- **Human capital** (*H*): The set of skills of the workers in the economy built through education and on-the-job training.
- The production function with human capital:

$$\frac{Y}{N} = f\left(\frac{K}{N}, \frac{H}{N}\right) \qquad (11.10)$$

$$(+, +)$$

- As for physical capital (*K*) accumulation, countries that save more or spend more on education can achieve higher steady-state levels of output per worker.

11-4 Physical versus Human Capital

- **Models of endogenous growth:** Steady-state growth in outpour per worker depends on variables such as the saving rate and the rate of spending on education, even without technological progress.
- However, the current consensus is that given the rate of technological progress, higher rates of saving or spending on education do not lead to a permanently higher growth rate.

Quick Check (ex. 1, p. 257)

1. Using the information in this chapter, label each of the following statements true, false, or uncertain. Explain briefly.

a. The saving rate is always equal to the investment rate.
b. A higher investment rate can sustain higher growth of output forever.
c. If capital never depreciated, growth could go on forever.
d. The higher the saving rate, the higher consumption in steady state.
e. A lower depreciation rate leads to an upward shift in the golden-rule level of capital.
f. Slower growth in the United States in the past 50 years could be attributed to a low savings rate only if the United States had started off with a much higher savings rate than today and there had been a substantial decline in the savings rate over time.
g. According to endogenous growth theory, higher levels of education lead to higher output growth rates.

Quick Check (ex. 1, p. 257)

1. Using the information in this chapter, label each of the following statements true, false, or uncertain. Explain briefly.

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f. Slower growth in the United States in the past 50 years could be attributed to a low savings rate only if the United States had started off with a much higher savings rate than today and there had been a substantial decline in the savings rate over time.

g. According to endogenous growth theory, higher levels of education lead to higher output growth rates.

1. a. True, if saving includes public and private saving.

b. False.

c. True. In the model without depreciation, there is no steady state. A constant saving rate produces a positive but declining rate of growth. In the infinite-time limit, the growth rate equals zero. Output per worker rises forever without bound. In the model with depreciation, if the economy begins with a level of capital per worker below the steady-state level, a constant saving rate also produces a positive but declining rate of growth, with a limit of zero. In this case, however, output per worker approaches a fixed number, defined by the steady-state condition of the Solow model. Note that depreciation is not needed to define a steady state if the model includes labor force growth or technological progress.

d. Uncertain. See the discussion of the golden-rule saving rate.

e. Uncertain/False. It is likely the U.S. rate is below the golden rule rate and that transforming Social Security to a pay-as-you-go system would ultimately increase the U.S. saving rate. These premises imply that such a transformation would increase U.S. consumption in the future, but not necessarily in the present. Indeed, if the only effect of such a transformation is to increase the saving rate, we know that consumption per worker will fall in the short run. Moreover, moving to a pay-as-you-go system requires transition costs. If these costs are borrowed, then the reduction in public saving will offset the increase in private saving during the transition. If these costs are not borrowed, then transitional generations must suffer either a reduction in promised benefits or an increase in taxes to finance their own retirement (at least to some degree) in addition to the retirement of a previous generation. Thus, whether the U.S. "should" move to a pay-as-you-go system depends on the likely resolution of intergenerational distributional issues and your view about the equity of such a resolution.

f. Uncertain. The U.S. capital stock is below the golden rule, but that does not necessarily imply that there should be tax breaks for saving. Even if the tax breaks were effective in stimulating saving, the increase in future consumption would come at the cost of current consumption.

h. False. Even if you accept the premise (that educational investment increases output, as would be implied by the Mankiw, Romer, Weil paper), it does not necessarily follow that countries should increase educational saving, since future increases in output will come at the expense of current consumption. Of course, there are other arguments for subsidizing education, particularly for low-income households.

APPENDIX: The Cobb-Douglas Production Function and the Steady State

- The Cobb-Douglas production function:

$$Y = K^{\alpha}N^{1-\alpha} \qquad (11.A1)$$

  which gives a good description of the relation between output, physical capital, and labor in the United States from 1899 to 1922.

- In steady state, saving per worker must be equal to depreciation per worker, implying that:

$$s(K^*/N)^{\alpha} = \delta(K^*/N)$$

  where K^* is the steady-state level of capital.

APPENDIX: The Cobb-Douglas Production Function and the Steady State

- The preceding expression can be rewritten as:

$$s = \delta(K^*/N)^{1-\alpha}$$

- The steady-state level of capital per worker becomes:

$$(K^*/N) = (s/\delta)^{\alpha/(1-\alpha)}$$

- If $\alpha = 0.5$, then:

$$K^*/N = s/\delta$$

  which implies that a doubling of the saving rate leads to a doubling in steady-state output per worker.